Sample Space Reducing process and the emergence of arbitrary scaling exponents

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Path dependence, non-stable phase spaces and the emergence of scaling patterns are key aspects of statistical physics of non-equilibrium systems. Sample Space Reducing processes (SSRps) are minimally path dependent stochastic processes which reduce their sample space as they unfold [1]. In addition, SSRps offer a new mechanism to understand the emergence of scaling. Therefore, they define a privileged framework to explore the above mentioned aspects of non-equilibrium systems in countless processes. In the simplest SSRps power law exponents can be related to noise levels in the process. Nevertheless, the emergence of scaling is not limited to the simplest SSRPs, but holds for a huge domain of stochastic processes that are characterised by non-uniform prior distributions. In the absence of noise the scaling exponents converge to a universal -1 exponent (Zipf's law) for almost all prior distributions [2]. As a consequence one can understand targeted diffusion on networks and its associated scaling laws in node visit distributions. The emergence of Zipfs law in node visiting statistics is expected in any random processes with a target in a network as, for example, a search, regardless the topological details of the graph [2].

SSRps theory is generalized by SSR cascading process, able to produce power laws with arbitrary exponents [3]. It can be shown analytically that the frequency distributions of states are power laws with exponents that coincide with the multiplication parameter of the cascading process. In addition, imposing energy conservation in SSR cascades, i.e., mimicking a desintegration/fragmentation process, allows us to recover an universal exponent -2, which is independent of the multiplication parameters of the cascade. This matches, for example, with Fermis classic result on the energy spectrum of cosmic rays.

The domain of application of SSRps and SSR cascades is huge, and includes general theory of dissipative systems, search patterns towards targets, traffic-, transport- and supply chain management, fragmentation processes or cascading processes on networks, such as rumour or epidemic spreading.

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